# Cross Sections and Collision Times

Weather you are talking about atoms in a gas, stars in a star cluster, or galaxies in intergalactic space, eventually some members will collide with each other. Predicting the collision times for various systems is an important way to estimate important events in their history.

*Image: Seyfert's Sextet galaxy cluster showing collisions (Courtesy Hubble Space Telescope)*

**Problem 1** - **Cross Sectional Area:** Draw two circles on a paper that overlap. These represent the geometrical cross sectional areas of two bodies. For example, the cross sectional area of a marble with a radius of 5 millimeters is A = $\pi$ (5mm)$^2$ = 78.5 mm$^2$. Find the cross sectional areas, in square meters, of: A) an oxygen atom with a radius of 50 picometers; B) a star with a radius of 698,000 km (in square kilometers); C) a galaxy with a radius of 50,000 light years (in square light years). (1 ly = 9.4 trillion km)

**Problem 2** - **Swept out volume:** A moving body sweeps out a cylindrical volume whose base area equals the bodies cross sectional area, and whose height equals the speed of the body times the elapsed time. Calculate the cylindrical volumes, in cubic meters, for; A) an atom of oxygen traveling at 500 meters/sec for 10 seconds; B) a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years (in cubic light years); C) the Milky Way galaxy traveling at 200 km/sec for 100 million years (in cubic light years). (1 year = 31 million seconds)

**Problem 3** - **Average particle volume:** The average volume occupied by a body in a system depends on the volume of the system and the number of bodies present. The 'number density' measures the number of particles divided by the volume of the system. The inverse of this number is the average volume per particle. Calculate the average volume for the following bodies: A) Sea-level atmosphere with 3 x 10$^{25}$ atoms/cubic meter; B) Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars (in stars per cubic light year) ; C) The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (in galaxies per cubic mega light year).

**Problem 4** - **Collision time:** The collision time is the time it takes a body to sweep out the same volume as is occupied by an average particle in the system. It is given by the formula T = 1/(N A V) where N is the density of particles, V is their average speed, and A is their average cross-sectional area. (All units should be in terms of meters and seconds.) From the area calculated in Problem 1, the speeds given in Problem 2, and the average particle densities from Problem 3, compute the particle collision times for A) an oxygen atom at sea level (in nanoseconds); B) a star in the Omega Centauri cluster (in years) and C) a galaxy in the Local Group (in years).

**Inquiry Question:** There are many more stars than galaxies, so why is it that galaxies collide nearly a million times more often?

# Answer Key

**Problem 1** - **A)** an oxygen atom with a radius of 50 picometers; $A = \pi (50 \times 10^{-12}\ m)^2$ = **$7.9 \times 10^{-21}$ meters$^2$** **B)** a star with a radius of 698,000 km; $A = \pi (698{,}000\ km)^2$ = **$1.5 \times 10^{12}$ km$^2$** **C)** a galaxy with a radius of 50,000 light years. $A = \pi(50{,}000\ ly)^2$ = **$7.9 \times 10^{9}$ ly$^2$**

**Problem 2** - Use the cross sectional areas calculated in Problem 1 and Volume = area x distance traveled; **A)** an atom of oxygen traveling at 500 meters/sec for 10 seconds; L = 500 m/s x 10 s = 5000 meters. Volume = $7.9 \times 10^{-21}$ meters$^2$ x 5000 meters = **$4.0 \times 10^{-17}$ meters$^3$** **B)** a star in the Omega Centauri cluster traveling at 22 km/sec for 1 million years; L = 22 km/s x 1 million yrs x 31 million sec/yr = $6.8 \times 10^{14}$ kilometers. Volume = $1.5 \times 10^{12}$ km$^2$ x $6.8 \times 10^{14}$ km = **$1.0 \times 10^{27}$ km$^3$**. **C)** the Milky Way galaxy traveling at 200 km/sec for 100 million years. L = 200 km/s x 100 million years x 31 million sec/yr = $6.2 \times 10^{17}$ kilometers = 66,000 light years. Volume = $7.9 \times 10^{9}$ ly$^2$ x $6.6 \times 10^{4}$ ly = **$5.2 \times 10^{14}$ ly$^3$**.

**Problem 3 -** Calculate the average volume for the following bodies: **A)** Sea-level atmosphere with $3 \times 10^{25}$ atoms/cubic meter; The number density, N = $3 \times 10^{25}$ atoms/meter$^3$ so 1/N = V = **$3.3 \times 10^{-26}$ meters$^3$ per atom**. **B)** Omega Centauri cluster with a diameter of 160 light years and containing 10 million stars; Assume the cluster is a sphere with a volume V = 4/3 $\pi$ (80 ly)$^3$ = $2.1 \times 10^{6}$ ly$^3$. Then N = 10 million / $2.1 \times 10^{6}$ ly$^3$ = 4.8 stars/ly$^3$. The individual star volume = 1/N = **0.2 ly$^3$ per star**. **C)** The Local Group of galaxies containing 35 galaxies including the Milky Way, with a diameter of 10 million light years (i.e. 10 Mly). Volume = 4/3 $\pi$ (5 Mly)$^3$ = 523 Mly$^3$ . N = 35 galaxies/ 523 Mly$^3$ = 0.07 galaxies/Mly$^3$ . Then 1/N = **14 Mly$^3$ per galaxy**.

**Problem 4 - A) Atom collision time:** A = $7.9 \times 10^{-21}$ meters$^2$ V = 500 m/sec and N = $3 \times 10^{25}$ atoms/m$^3$. so T = 1/($7.9 \times 10^{-21}$ meters$^2$ x 500 m/sec x $3 \times 10^{25}$ atoms/m$^3$) = **$8.4 \times 10^{-9}$ seconds or about 8 nanoseconds**.

**B) Star collision time**: A = $1.5 \times 10^{12}$ km$^2$ V = 22 km/sec and N = 4.8 stars/Ly$^3$ . converting them to the same units, in this case kilometers, N = 4.8/($9.4 \times 10^{12}$)$^3$ = $5.8 \times 10^{-39}$, so T = 1/(1.5 x $10^{12}$ x 22 km/sec x $5.8 \times 10^{-39}$) = $5.2 \times 10^{24}$ seconds or **$1.7 \times 10^{17}$ years**.

**C) Galaxy collision time**: A = $7.9 \times 10^{9}$ ly$^2$ ; V = 200 km/sec, which is equal to 200 km/sec x (1 Ly/$9.4 \times 10^{12}$ km) x (31 million seconds/1 year) = 0.0066 Ly/year; and N = 0.07 galaxies/Mly$^3$ which is $0.07 \times 10^{-18}$ galaxies/ly$^3$; so T = 1/( $7.9 \times 10^{9}$ x 0.0066 x $7.0 \times 10^{-20}$) = **$2.7 \times 10^{11}$ years.**

**Inquiry Question:** Students should recognize that, although stars and galaxies travel about the same speed, galaxies have much larger cross sections compared to the volume of space they occupy, so there will be more collisions between them.